

Exam Calculus 1

8 februari 2011, 9.00-12.00.

This exam has 7 problems. The maximum score per problem can be found below. Write on each page your name and student number, and on the first page your seminar group. The use of annotations, books and calculators is not permitted in this examination. All answers must be supported by arguments and/or work. Success.

- (a) Formulate the principle of mathematical induction.
(b) Prove that if $n \geq 1$ is a positive integer, then $x^n - 1$ is divisible by $x - 1$ (where $x \neq 1$).

- Find all (complex) solutions of

$$z^5 = 2\sqrt{3} + 2i$$

and plot them in the complex plane.

- (a) The function $f(x)$ is defined on $-\infty < x < \infty$. Give the ϵ - δ -definition of

$$\lim_{x \rightarrow a} f(x) = L.$$

- (b) Prove (with the help of this definition) that

$$\lim_{x \rightarrow 3} (4x - 5) = 7$$

- Newton's law of Gravitation says that the magnitude F of the force exerted by the earth on a body with unit mass at a distance r from the center of the earth is given by

$$F(r) = \begin{cases} GMr/R^3 & \text{if } r < R \\ GM/r^2 & \text{if } r \geq R \end{cases}$$

where R denotes the radius of the earth, M is the mass of the earth and G is the gravitational constant. (Note: R , M and G are constant).

- (a) Is the function $F(r)$ continuous?
(b) Is F differentiable at $r = R$?

5. (a) Find the derivative of

$$f(x) = (\sin x)^{\ln x}$$

(b) Idem,

$$f(x) = \int_{-2}^{3x} \sin(2t) dt$$

6. (a) Evaluate

$$\int \frac{e^x}{e^x + 1} dx$$

(b) Evaluate

$$\int_0^{\ln 2} \frac{1}{e^x + 1} dx$$

(c) Evaluate

$$\int_1^2 x^2 \ln(6x) dx$$

7. Take $t > 0$. Find the solution $y(t)$ of the differential equation

$$t \frac{dy}{dt} + 2y = t^3$$

that satisfies $y(1) = 0$.

Maximum score:

1a	0.5	2	1.0	3a	0.7	4a	0.7	5a	0.5	6a	0.5	7	1.5
b	0.5			b	0.8	b	0.8	b	0.5	b	0.5		
										c	0.5		